Texas Vegetable Remote Sensing Study, 1969

Determination of Optimum Plot Size and Shape for Estimation of Carrot Yield

by

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Abstract

Counts and measurements made during a remote sensing experiment indicated that total weight and number of plants per plot are observations highly correlated with yield. The data on Texas carrots show that plant height and carrot length for carrots subsampled within plots are closely related to yield. Crown circumference and carrot length are directly related to individual carrot weight. The optimum plot for estimating yield from both number of plants and total weight per plot is a one bed plot three to five feet long. Optimum plot size and shape were determined by both a discrete and a continuous procedure.

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I. Introduction

This analysis is based upon data collected in January 1969 from two selected carrot fields located in the Lower Rio Grande Valley of Texas. The data was collected primarily to study relationships with remote sensed data. Each field contained five randomly located sample plots, each of which contained three beds 1/ nine feet in length. Each bed was in turn divided into three subplots, 1 bed x 3 feet in size (See Figure 1). Number of plants and total weight of harvested carrots were the observations obtained for each 1x3' plot. In addition, the weight, height, and crown circumference for two randomly selected carrots were obtained for each subplot 2/.

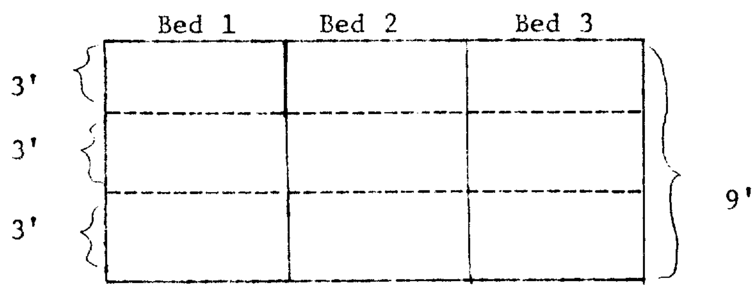

Figure 1

1/ A bed is defined to be the row or rows of a crop between two irrigation ditches.

2/ Length of the two carrots for each subplot was also obtained in one field.

II. Objectives

A primary objective of this study was to obtain information needed for planning remote sensing research on carrot yield. Remote Sensing is directed toward determining the relationship between remote sensed data and "important" ground data. For crops, remote sensed data is usually obtained by aerial photography and converted to numeric data by making various optical density [1/] readings from the photographs. To plan such research, what ground data are "important" needs to be known. An indication of the importance of various types of ground data can be obtained by studying the correlation of the various observations with crop yield. Also, it is desirable to know the optimum plot size for the "important" ground data. If remote sensing requirements permit, the optimum plot can be used so that the variance of estimates from the ground data are near the minumum for a given cost.

An additional objective was to obtain information about carrot plant characteristics. Very little sampling data on carrots are available. Correlation analyses of the data on a per carrot basis and for various plot sizes are presented. The nested analyses of variance provides estimates of variances for the various observations on different plot sizes.

Another purpose was the development of procedures for optimum plot determination. Two procedures can be used to determine the optimum plot size and shape. One procedure is to consider plot size as a discrete variable and select the optimum of the plots studied. Nested analyses of variance are used to estimate the nested components of variance. The estimated independent mean squares and cost estimates are used to determine the plot with minimum variance for a given cost. Another procedure is to consider plot size as a continuous variable. The variance of a plot is assumed to be a function of plot size. There are several alternative methods of fitting a function of variance in terms of plot size. A combination of the discrete and continuous procedure involves determining either the optimum length or width by the discrete procedure and then using the continuous procedure to determine the remaining dimension of the optimum plot.

1/ Optical density is the common logarithum of the ratio of the intensities of the light incident upon, to light transmitted through a material.

III. Correlation Analyses of Plant Characteristics

In determining what ground observations are related to carrot yield, correlation analyses can be made on an individual carrot basis and for various plot sizes. The correlation coefficients on a per carrot basis are shown in Table 1 for each field.

Table 1.--Correlation matrix per carrot - by fields

	Weight	Height	Crown circumference	Length 1/
		Field A		
Weight	1.000			
Height	.153	1.000		
Crown circumference	.932**	.097	1.000	----
		Field B		
Weight	1.000			
Height	.007	1.000		
Crown circumference	.648**	.006	1.000	
Length	.789**	-.015	.424**	1.000

1/ Length of carrots was measured only in Field B

* Significantly different from zero at the 5% level
** Significantly different from zero at the 1% level

The correlation analysis on a per carrot basis indicates crown circumference and carrot length are related to individual carrot yield (weight).

A correlation analysis was made on a 1 bed x 3 foot plot basis between measurements of the average height, average crown circumference and average length of two carrots, number of plants, and plot weight. The correlations on a 1x3' plot basis are shown in Table 2.

Table 2.--Correlation matrix per 1x3' plot - by fields

	Plot weight	Number of plants	Average for two carrots subsampled per plot 1/		
			Height	Crown circumference	Length 2/
				Field A	
Plot weight	1.000				
Number of plants	.883**	1.000			
Average height	.746**	.553**	1.000		
Average crown circumference	-.106	-.313*	.123	1.000	-----
				Field B	
Plot weight	1.000				
Number of plants	.907**	1.000			
Average height	.760**	.779**	1.000		
Average crown circumference	-.094	-.178	.003	1.000	
Average length	-.285	-.459**	-.292	.501**	1.000

1/ The effects of errors in measurements at the plot level due to subsampling reduces the expected correlation coefficient. Consequently, the coefficient is understated for variables which were subsampled within plots.

2/ Length of carrots was measured only in Field B.

* Significantly different from zero at the 5% level
** Significantly different from zero at the 1% level

Number of plants and average height of two carrots subsampled per 1x3' plot were the only observations strongly related to plot weight. Correlations such as these are, of course, in part dependent upon the sampling rate of two carrots per 1x3' plot.

Correlations based upon the 3 bed x 9 foot plot are shown in Table 3.

Table 3.--Correlation matrix per 3x9' plot - by fields

	Plot weight	Number of plants	Average for 18 carrots subsampled at the rate of two per 1x3' plot 1/ Height	Crown circumference	Length 2/
				Field A	
Plot weight	1.000				
Number of plants	.765	1.000			
Average height	.848	.423	1.000		
Average crown circumference	.675	.150	.875	1.000	----
				Field B	
Plot weight	1.000				
Number of plants	.996**	1.000			
Average height	.995**	.999**	1.000		
Average crown circumference	-.309	-.366	-.398	1.000	
Average length	-.886*	-.917*	-.907*	.461	1.000

1/ The effects of errors in measurements at the plot level due to subsampling reduces the expected correlation coefficient. Consequently, the coefficient is understated for variables which were subsampled within plots.

2/ Length of carrots was measured only in Field B.

* Significantly different from zero at the 5% level
** Significantly different from zero at the 1% level

The correlation coefficients in Table 3 indicate number of plants, average height and average length are related to 3x9' plot carrot weight. Average height is related positively while average length is related negatively.

Correlation coefficients for total weight, number of plants, average weight of two carrots and estimated weight on a 1x3' plot basis are shown in Table 4.

Table 4.--Correlation matrix per 1x3' plot - for combined fields

	Plot weight	Number of plants	Average weight of two carrots per 1x3' plot	Estimated weight 1/
Plot weight	1.000			
Number of plants	.851**	1.000		
Average weight	-.194	-.427**	1.000	
Estimated weight 1/	.764**	.690**	-.204	1.000

1/ Estimated weight is number of plants x average weight of two carrots for each 1x3' plot

* Significantly different from zero at the 5% level
** Significantly different from zero at the 1% level

Table 4 shows that number of plants and estimated weight are related to yield per plot. Of course, the correlation coefficient of estimated weight with plot weight is in part dependent on the number of carrots subsampled per 1x3' plot. Note that average weight and number of plants has a highly significant negative correlation coefficient. For the 3x9' plot the correlation coefficient for plot weight and number of plants is .921. This is significant at the 1% level.

The strength of the relationships of various observations to yield is affected by the size of plot upon which the observations are made. If the optimum plot is expected to be greater than the 1x3' unit but less than the 3x9' unit, the important observations according to the correlation analyses are number of plants and total weight per plot.

Observations important on carrots subsampled within plots are plant height, carrot length and possibly some other observations easily obtained. The observations on the subsampled carrots can be obtained with sufficient reliability by taking the proper size of subsample within the plot. The optimum plot should, therefore, be selected so that number of plants and total weight are estimated with minimum variance for a given cost.

IV. Optimum Plot Selection from Eight Plot Sizes and Shapes

Optimum plot size and shape is considered as a discrete variable in two dimensions in the following analysis. Plots 1 bed x 3 feet (a), 1 bed x 6 feet (b), 1 bed x 9 feet (c), 2 beds x 3 feet (d), 2 beds x 9 feet (e), 3 beds x 3 feet (g), 3 beds x 6 feet (h), and 3 beds x 9 feet (i) are studied. The letters in parentheses are used as subscripts to refer to plot size and shape in the tables. As indicated above, possible methods of estimating yield involve data on number of plants and total carrot weight for the entire plot. Therefore, both number of plants and total weight are the criteria for optimum plot selection used in this analysis.

1. Analysis of 3 bed x 9 foot Plots

Nested analyses of variance for the ten 3 bed x 9 foot plots are shown below for number of plants and total weight. Two analyses are given for each of these. The first analysis is based upon the 1x3' plot within the 1x9' plot within the 3x9' plot and the second is based upon the 1x3' plot within the 3x3' plot within the 3x9' plot. Note that K_f^2 is used instead of σ_f^2 since the fields were not selected randomly.

Number of plants-analysis of variance number 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	3,475.19	3,475.19	$\sigma^2 a/c + 3\sigma^2 c/i + 9\sigma^2 i/f + 45K_f^2$
3/9'/f	8	12,963.36	1,620.42	$\sigma^2 a/c + 3\sigma^2 c/i + 9\sigma^2 i/f$
1/9'/3x9'	20	2,927.11	146.36	$\sigma^2 a/c + 3\sigma^2 c/i$
1x3'/1x9'	60	4,849.33	80.82	$\sigma^2 a/c$
Total	89	24,214.99		

Number of plants - analysis of variance number 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	3,475.19	3,475.19	$\sigma^2 a/g + 3\sigma^2 g/i + 9\sigma^2 i/f + 45K_f^2$
3x9'/f	8	12,963.36	1,620.42	$\sigma^2 a/g + 3\sigma^2 g/i + 9\sigma^2 i/f$
3x3'/3x9'	20	1,727.11	86.36	$\sigma^2 a/g + 3\sigma^2 g/i$
1x3'/3x3'	60	6,049.33	100.82	$\sigma^2 a/g$
Total	89	24,214.99		

Total weight-analysis of variance number 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	2.717	2.717	$\sigma^2 a/c + 3\sigma^2 c/i + 9\sigma^2 i/f + 45K_f^2$
3x9'/f	8	76.927	9.616	$\sigma^2 a/c + 3\sigma^2 c/i + 9\sigma^2 i/f$
1x9'/3x9'	20	31.258	1.563	$\sigma^2 a/c + 3\sigma^2 c/i$
1x3'/1x9'	60	35.137	0.586	$\sigma^2 a/c$
Total	89	146.039		

Total weight-analysis of variance number 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	2.717	2.717	$\sigma^2 a/g + 3\sigma^2 g/i + 9\sigma^2 i/f + 45K_f^2$
3x9'/f	8	76.927	9.616	$\sigma^2 a/g + 3\sigma^2 g/i + 9\sigma^2 i/f$
3x3'/3x9'	20	17.851	0.893	$\sigma^2 a/g + 3\sigma^2 g/i$
1x3'/3x3	60	48.544	0.809	$\sigma^2 a/g$
Total	89	146.039		

To make valid comparisons between the various size plots, it is necessary to obtain estimates of their variances when the population consists only of plots of a specific size. That is, estimates of the independent mean squares for a population of given size units are needed. The estimated variances of the units smaller than 3x9' are not the mean squares in the analysis of variance table since these smaller units are not a simple random sample from the population of units. These estimates are biased because the sampled units are in contiguous groups of 3 and 9 units. All estimates of variance are presented in terms of 1x3' plots so that comparisons can be made. The independent mean squares are derived from the nested components by the following equations:

$$\hat{\sigma}^2_{1x3'} = \hat{\sigma}^2_{a/c} + \hat{\sigma}^2_{c/i} + \hat{\sigma}^2_{i/f}$$

$$\hat{\sigma}^2_{1x9'} = \frac{\hat{\sigma}^2_{a/c}}{3} + \hat{\sigma}^2_{c/i} + \hat{\sigma}^2_{i/f}$$

$$\hat{\sigma}^2_{3x9'} = \frac{\hat{\sigma}^2_{a/c}}{9} + \frac{\hat{\sigma}^2_{c/i}}{3} + \hat{\sigma}^2_{i/f} \text{ from analysis 1, and}$$

$$\hat{\sigma}^2_{1x3'} = \hat{\sigma}^2_{a/g} + \hat{\sigma}^2_{g/i} + \hat{\sigma}^2_{i/f}$$

$$\hat{\sigma}^2_{3x3'} = \frac{\hat{\sigma}^2_{a/g}}{3} + \hat{\sigma}^2_{g/i} + \hat{\sigma}^2_{i/f}$$

$$\hat{\sigma}^2_{3x9'} = \frac{\hat{\sigma}^2_{a/g}}{9} + \frac{\hat{\sigma}^2_{g/i}}{3} + \hat{\sigma}^2_{i/f} \text{ from analysis 2.}$$

Estimated variances within fields are shown in Tables 5 and 6. Note that the common independent mean squares have the same estimate from analysis 1 and 2.

Table 5.--Estimated variance of number of plants within fields

Analysis	Nested component	Estimate	Independent mean square	Estimate 2/
Number 1	$\sigma^2 a/c$	80.82	$\sigma^2 a$	266.45
	$\sigma^2 c/i$	21.85	$\sigma^2 c$	212.57
	$\sigma^2 i/f$	163.78	$\sigma^2 i$	180.04
Number 2	$\sigma^2 a/g$	100.82	$\sigma^2 a$	266.45
	$\sigma^2 g/i$	-4.82 1/	$\sigma^2 g$	199.24
	$\sigma^2 i/f$	170.45	$\sigma^2 i$	180.04

1/ It is customary to set negative variances equal to zero. Here the negative estimate is important in reflecting that there is very little variation between three foot sections of 3x9' plots.

2/ Mean squares are adjusted to the 1 bed x 3 foot plot level.

Table 6.--Estimated variance of total weight within fields

Analysis	Nested component	Estimate	Independent mean square	Estimate 2/
Number 1	$\sigma^2 a/c$	0.586	$\sigma^2 a$	1.81
	$\sigma^2 c/i$	0.326	$\sigma^2 c$	1.42
	$\sigma^2 i/f$	0.895	$\sigma^2 i$	1.07
Number 2	$\sigma^2 a/g$	0.809	$\sigma^2 a$	1.81
	$\sigma^2 g/i$	0.028	$\sigma^2 g$	1.27
	$\sigma^2 i/f$	0.969	$\sigma^2 i$	1.07

2/ Mean squares are adjusted to the 1 bed x 3 foot plot level.

The independent mean squares can be considered relative to the variance of the 1x3' plot. The ratio of the variance of a plot to the variance of the 1x3' plot indicates the number of plots of that size required per field to give as precise a field estimate as a single 1x3' plot. The ratios are shown in Table 7.

Table 7.--Ratio of estimated variances to variance of the 1x3' plot

Plot	Ratio	Number of plants	Total weight
1x3'	$\hat{\sigma}^2_a/\hat{\sigma}^2_a$	1.00	1.00
1x9'	$\hat{\sigma}^2_c/\hat{\sigma}^2_a$	0.80	0.78
3x3'	$\hat{\sigma}^2_g/\hat{\sigma}^2_a$	0.75	0.70
3x9'	$\hat{\sigma}^2_i/\hat{\sigma}^2_a$	0.68	0.59

2. <u>The Cost Function</u>

Further determination of optimum plot size and shape depends upon the cost per plot of obtaining data for each size of plot. Relative cost variances can be used for this purpose. The ratio of the variance of a plot (whatever size) to the variance of a 1x3' plot multiplied by the ratio of the cost of the plot to the cost of a 1x3' plot, $\frac{\hat{\sigma}^2_P}{\hat{\sigma}^2_a} \times \frac{C_P}{C_a}$, is the relative cost variance. The plot with the lowest relative cost variance is the optimum plot size and shape. That is, it will give an estimate of maximum precision for a given cost or the minimum cost for a given level of precision.

Symbols will be used extensively in the following discussion of cost because estimates of cost are not as reliable as are estimates of variance. A suggested model for the cost per plot for each visit is:

$$C_P = \frac{C_L}{V} + C_B + C_W = W\left(\frac{T_L}{V} + T_B + T_W\right);$$

where C_P = cost per plot, C_L = one-time cost of randomly locating and defining boundaries of a plot, V = number of visits, C_B = cost between plots, C_W = cost within plots, W = wage per minute for one enumerator and the T's indicate the corresponding costs in terms of time (minutes). If T_L is divided into a component independent of plot size, time to randomly locate the plot (T_R) and a component of time to define plot boundaries (T_D), then we have

$$C_P = W\left(\frac{T_R + T_D}{V} + T_B + T_W\right)$$

Here, T_R, V, and T_B are constants with respect to plot size. Time between plots (T_B) is assumed to be constant with respect to plot size because for relatively small fields average distance between plots is quite uniform within a limited range for the number of plots per field. Since the definition of boundaries of a plot involves measuring ℓ feet along the bed from the starting corner and then defining ends of the unit across w beds, $T_D = .3\sqrt{\ell} + 2\sqrt{w}$ is perhaps a reasonable model for time required to define a plot. The time to collect data within a plot (T_W) is composed of the time to gather data[1/] within ℓ distance on each of w beds. A suggested model is $T_W = 5\,w\sqrt{\ell}$. Thus, the model for cost per plot for each visit is:

$$(1)\quad C_P = W\left(\frac{T_R + .3\sqrt{\ell} + 2\sqrt{w}}{V} + T_B + 5\,w\sqrt{\ell}\right),$$

1/ Data to be gathered is assumed to include a reasonable number of observations within the plot. Observations might be made for many characteristics, each requiring little time, but together time should be allowed for them.

where W, T_R, V and T_B are independent of plot size. This cost function is based upon limited information, but its form agrees with available data. For further analysis of cost we must now assume values for T_R, V and T_B. If T_R = 10.00, V = 3 and T_B = 2.00, then C_a = 14.82W, C_c = 21.30W, C_g = 32.61W and C_i = 51.79W 1/. Thus,

$$\frac{C_a}{C_a} = 1.00, \quad \frac{C_c}{C_a} = \frac{21.30W}{14.82W} = 1.44, \quad \frac{C_g}{C_a} = \frac{32.61W}{14.82W} = 2.20$$

and $\frac{C_i}{C_a} = \frac{51.79W}{14.82W} = 3.49$. The resulting relative cost variances are shown in the following table.

Table 8.--Relative cost variances for four plot sizes and shapes

Plot	Relative cost variances	Number of plants	Total weight
1x3'	$\hat{\sigma}^2_a/\hat{\sigma}^2_a \times C_a/C_a$	1.00	1.00
1x9'	$\hat{\sigma}^2_c/\hat{\sigma}^2_a \times C_c/C_a$	1.15	1.12
3x3'	$\hat{\sigma}^2_g/\hat{\sigma}^2_a \times C_g/C_a$	1.65	1.54
3x9'	$\hat{\sigma}^2_i/\hat{\sigma}^2_a \times C_i/C_a$	2.37	2.06

1/ No assumption is necessary for W, but for an indication of costs in dollars, it can be assumed at 4 1/3 cents/minute or $2.60/hour. Thus, C_a = \$0.64, C_c = \$0.92, C_g = \$1.41, and C_i = \$2.24.

Table 8 indicates that the 1x3' plot is the optimum plot size of these four plots for estimating number of plants and total weight.

3. Analysis of 3 Bed by 6 Foot Plots

Now, to consider other plot sizes, a 3 bed x 6 foot plot may be analyzed by alternately excluding 3 feet at either end of the 3x9' unit. This gives two sets of data upon which nested analyses of variance for the 3x6' units are shown below. Figure 2 shows which 1 bed x 3 foot plots are included in each data set. The analyses for each of the sets of data, denoted set 1 and set 2, are presented in two ways. In analyses 3 and 5 the 1x3' plot is taken within the 1x6' within the 3x6' plot and in analyses 4 and 6 the 1x3' plot is taken within the 3x3' plot within the 3x6' plot.

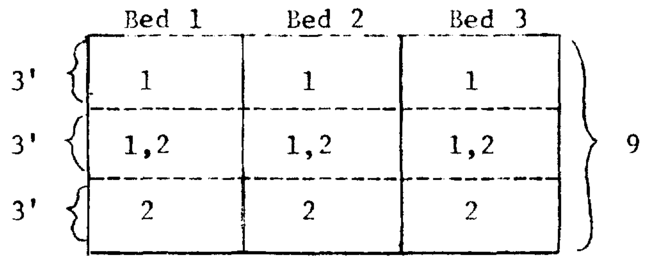

Figure 2

Number of plants - analysis of variance number 3 - data set 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean squares
Fields (f)	1	2,856.60	2,856.60	$\sigma^2 a/b + 2\sigma^2 b/h + 6\sigma^2 h/f + 30K_f^2$
3x6'/F	8	10,169.66	1,271.21	$\sigma^2 a/b + 2\sigma^2 b/h + 6\sigma^2 h/f$
1x6'/3x6'	20	1,956.70	97.84	$\sigma^2 a/b + 2\sigma^2 b/h$
1x3'/1x6'	30	2,138.97	71.30	$\sigma^2 a/b$
Total	59	17,121.93		

Number of plants - analysis of variance number 4 - data set 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	2,856.60	2,856.60	$\sigma^2 a/g + 3\sigma^2 g/h + 6\sigma^2 h/f + 30K_f^2$
3x6'/f	8	10,169.66	1,271.21	$\sigma^2 a/g + 3\sigma^2 g/h + 6\sigma^2 h/f$
3x3'/3x6'	10	931.10	93.10	$\sigma^2 a/g + 3\sigma^2 g/h$
1x3'/3x3'	40	3,164.67	79.12	$\sigma^2 a/g$
Total	59	17,121.93		

Number of plants - analysis of variance number 5 - data set 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	2,760.81	2,760.81	$\sigma^2 a/b + 2\sigma^2 b/h + 6\sigma^2 h/f + 30K_f^2$
3x6'/f	8	9,688.00	1,211.00	$\sigma^2 a/b + 2\sigma^2 b/h + 6\sigma^2 h/f$
1x6'/3x6'	20	3,020.15	151.01	$\sigma^2 a/b + 2\sigma^2 b/h$
1x3'/1x6'	30	2,770.02	92.33	$\sigma^2 a/b$
Total	59	18,238.98		

Number of plants - analysis of variance number 6 - data set 2

Source	Degree of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	2,760.81	2,760.81	$\sigma^2 a/g + 3\sigma^2 g/h + 6\sigma^2 h/f + 30K_f^2$
3x6'/f	8	9,688.00	1,211.00	$\sigma^2 a/g + 3\sigma^2 g/h + 6\sigma^2 h/f$
3x3'/3x6'	10	1,050.84	105.08	$\sigma^2 a/g + 3\sigma^2 g/h$
1x3'/3x3'	40	4,739.33	118.48	$\sigma^2 a/g$
Total	59	18,238.98		

Total weight - analysis of variance number 3 - data set 1

Source	Degree of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	6.882	6.882	$\sigma^2 a/b + 2\sigma^2 b/h + 6\sigma^2 h/f + 30K_f^2$
3x6'/f	8	50.030	6.254	$\sigma^2 a/b + 2\sigma^2 b/h + 6\sigma^2 h/f$
1x6'/3x6'	20	2.279	0.114	$\sigma^2 a/b + 2\sigma^2 b/h$
1x3'/1x6'	30	38.910	1.297	$\sigma^2 a/b$
Total	59	98.101		

Total weight - analysis of variance number 4 - data set 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	6.882	6.882	$\sigma^2 a/g + 3\sigma^2 g/h + 6\sigma^2 h/f + 30K_f^2$
3x6'/f	8	50.030	6.254	$\sigma^2 a/g + 3\sigma^2 g/h + 6\sigma^2 h/f$
3x3'/3x6'	10	8.545	0.854	$\sigma^2 a/g + 3\sigma^2 g/h$
1x3'/3x3'	40	32.644	0.816	$\sigma^2 a/g$
Total	59	98.101		

Total weight - analysis of variance number 5 - data set 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	2.098	2.098	$\sigma^2 a/b + 2\sigma^2 b/h + 6\sigma^2 h/f + 30K_f^2$
3x6'/f	8	60.893	7.612	$\sigma^2 a/b + 2\sigma^2 b/h + 6\sigma^2 h/f$
1x6'/3x6'	20	23.325	1.166	$\sigma^2 a/b + 2\sigma^2 b/h$
1x3'/1x6'	30	17.681	0.589	$\sigma^2 a/b$
Total	59	103.997		

Total weight - analysis of variance number 6 - data set 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	2.098	2.098	$\sigma^2 a/g + 3\sigma^2 g/h + 6\sigma^2 h/f + 30 K_f^2$
3x6'/f	8	60.893	7.612	$\sigma^2 a/g + 3\sigma^2 g/h + 6\sigma^2 h/f$
3x3'/3x6'	10	9.453	0.945	$\sigma^2 a/g + 3\sigma^2 g/h$
1x3'/3x3'	40	31.553	0.789	$\sigma^2 a/g$
Total	59	103.997		

The independent mean squares in terms of the 1x3' plots are derived from the nested components by the following equations:

$$\hat{\sigma}^2 1x3' = \hat{\sigma}^2 a/b + \hat{\sigma}^2 b/h + \hat{\sigma}^2 h/f$$

$$\hat{\sigma}^2 1x6' = \frac{\hat{\sigma}^2 a/b}{2} + \hat{\sigma}^2 b/h + \hat{\sigma}^2 h/f$$

$$\hat{\sigma}^2 3x6' = \frac{\hat{\sigma}^2 a/b}{2} + \frac{\hat{\sigma}^2 b/h}{3} + \hat{\sigma}^2 h/f$$ for analyses 3 and 5, and

$$\hat{\sigma}^2 1x3' = \hat{\sigma}^2 a/g + \hat{\sigma}^2 g/h + \hat{\sigma}^2 h/f$$

$$\hat{\sigma}^2 3x3' = \frac{\hat{\sigma}^2 a/g}{3} + \hat{\sigma}^2 g/h + \hat{\sigma}^2 h/f$$

$$\hat{\sigma}^2 3x6' = \frac{\hat{\sigma}^2 a/g}{6} + \frac{\hat{\sigma}^2 g/h}{2} + \hat{\sigma}^2 h/f$$ for analyses 4 and 6.

Tables 9 and 10 show the estimated variances within fields.

Table 9.--Estimated variance of number of plants within fields

Analysis and data set	Nested component	Estimate	Independent mean square	Estimate[1/]
Number 3	$\sigma^2_{a/b}$	71.30	σ^2_a	280.13
	$\sigma^2_{b/h}$	13.27	σ^2_b	244.48
Data set 1	$\sigma^2_{h/f}$	195.56	σ^2_h	211.87
Number 4	$\sigma^2_{a/g}$	79.12	σ^2_a	280.13
	$\sigma^2_{g/h}$	4.66	σ^2_g	227.38
Data set 1	$\sigma^2_{h/f}$	196.35	σ^2_h	211.87
Number 5	$\sigma^2_{a/b}$	92.33	σ^2_a	298.33
	$\sigma^2_{b/h}$	29.34	σ^2_b	252.16
Data set 2	$\sigma^2_{h/f}$	176.66	σ^2_h	201.83
Number 6	$\sigma^2_{a/g}$	118.48	σ^2_a	298.33
	$\sigma^2_{g/h}$	-4.47	σ^2_g	228.28
Data set 2	$\sigma^2_{h/f}$	184.32	σ^2_h	201.83

1/ Mean squares are adjusted to the 1 bed x 3 foot plot level.

Table 10.--Estimated variance of total weight within fields

Analysis and data set	Nested component	Estimate	Independent mean square	Estimate[1/]
Number 3	$\sigma^2_{a/b}$	1.297	σ^2_a	1.73
	$\sigma^2_{b/h}$	-0.592	σ^2_b	1.08
Data set 1	$\sigma^2_{h/f}$	1.023	σ^2_h	1.04
Number 4	$\sigma^2_{a/g}$	0.816	σ^2_a	1.73
	$\sigma^2_{g/h}$	0.013	σ^2_g	1.18
Data set 1	$\sigma^2_{h/f}$	0.900	σ^2_h	1.04
Number 5	$\sigma^2_{a/b}$	0.589	σ^2_a	1.95
	$\sigma^2_{b/h}$	0.288	σ^2_b	1.66
Data set 2	$\sigma^2_{h/f}$	1.074	σ^2_h	1.27
Number 6	$\sigma^2_{a/g}$	0.789	σ^2_a	1.95
	$\sigma^2_{g/h}$	0.052	σ^2_g	1.43
Data set 2	$\sigma^2_{h/f}$	1.111	σ^2_h	1.27

1/ Mean squares are adjusted to the 1 bed x 3 foot plot level.

Since the difference between data set 1 and 2 is merely a distance of three feet along a bed in locating the corner of a 3x6' plot, it seems reasonable to average the two data sets' estimates of the independent mean squares to derive a best estimate. The derived estimates are shown in Table 11.

Table 11.--Estimated independent mean squares for four plot sizes and shapes

Plot	Independent mean square	Number of plants	Total weight
1x3'	σ^2_a	289.23	1.84
1x6'	σ^2_b	248.32	1.37
3x3'	σ^2_g	227.83	1.30
3x6'	σ^2_h	206.85	1.16

These independent mean squares can be considered relative to the variance of the 1x3' plot. The ratios are shown in Table 12.

Table 12.--Ratios of estimated variances to estimated variance of the 1x3' plot

Plot	Ratio	Number of plants	Total weight
1x3'	σ^2_a/σ^2_a	1.00	1.00
1x6'	σ^2_b/σ^2_a	0.86	0.74
3x3'	σ^2_g/σ^2_a	0.79	0.71
3x6'	σ^2_h/σ^2_a	0.72	0.63

By substitution into equation (1) on page 12, we have

$$C_{1x6'} = W\left(\frac{T_R + .3\sqrt{\ell} + 2\sqrt{w}}{V} + T_B + 5_w\sqrt{\ell}\right)$$

$$= W\left(\frac{10.00 + 2.74}{3} + 2.00 + 12.25\right) = 18.50\ W$$

and $$C_{3x6'} = W\left(\frac{10.00 + 4.20}{3} + 2.00 + 36.75\right) = 43.48\ W.$$

Thus, $\frac{C_a}{C_a} = 1.00$, $\frac{C_b}{C_a} = \frac{18.50W}{14.82W} = 1.25$, $\frac{C_g}{C_a} = \frac{32.61W}{14.82W} = 2.20$, and

$$\frac{C_h}{C_a} = \frac{43.48W}{14.82W} = 2.93.$$

The relative cost variances for these plots are shown in Table 13.

Table 13.--Relative cost variances for four plot sizes and shapes

Plot	Relative cost variances	Number of plants	Total weight
1x3'	$\hat{\sigma}^2_a/\hat{\sigma}^2_a \times C_a/C_a$	1.00	1.00
1x6'	$\hat{\sigma}^2_b/\hat{\sigma}^2_a \times C_b/C_a$	1.08	0.92
3x3'	$\hat{\sigma}^2_g/\hat{\sigma}^2_a \times C_g/C_a$	1.74	1.56
3x6'	$\hat{\sigma}^2_h/\hat{\sigma}^2_a \times C_h/C_a$	2.11	1.85

Table 13 indicates that the 1x3' plot is the optimum plot size of these four plots for estimating number of plants and that the 1x6' plot is the optimum plot for estimating total weight.

4. Analysis of 2 Bed by 9 Foot Plots

To consider the two additional plots, a 2 bed x 9 foot plot may be analyzed by excluding alternately one bed at either side of the 3x9' unit. For each of the data sets created, nested analyses of variance of the ten 2x9' units are shown. Figure 3 shows which 1 bed x 3 foot plots are included in each data set. The analyses for each set of data are presented in two ways. In analyses number 7 and 9, the 1x3' plot is taken within the 1x9' within the 2x9' plot, and in analyses number 8 and 10 the 1x3' plot is taken within the 2x3' within the 2x9' plot.

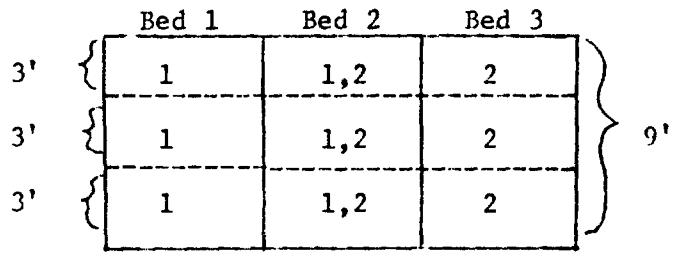

Figure 3

Number of plants - analysis of variance number 7 - data set 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	1,346.77	1,346.77	$\sigma^2_{a/c} + 3\sigma^2_{c/e} + 6\sigma^2_{e/f} + 30K_f^2$
2x9'/f	8	8,234.89	1,029.36	$\sigma^2_{a/c} + 3\sigma^2_{c/e} + 6\sigma^2_{e/f}$
1x9'/2x9'	10	1,021.85	102.18	$\sigma^2_{a/c} + 3\sigma^2_{c/e}$
1x3'/1x9'	40	4,014.82	100.37	$\sigma^2_{a/c}$
Total	59	14,618.33		

Number of plants - analysis of variance number 8 - data set 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	1,346.77	1,346.77	$\sigma^2 a/d + 2\sigma^2 d/e + 6\sigma^2 e/f + 30K_f^2$
2x9'/f	8	8,234.89	1,029.36	$\sigma^2 a/d + 2\sigma^2 d/e + 6\sigma^2 e/f$
2x3'/2x9'	20	2,300.67	115.03	$\sigma^2 a/d + 2\sigma^2 d/e$
1x3'/2x3'	30	2,736.00	91.20	$\sigma^2 a/d$
Total	59	14,618.33		

Number of plants - analysis of variance number 9 - data set 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	3,435.27	3,435.27	$\sigma^2 a/c + 3\sigma^2 c/e + 6\sigma^2 e/f + 30K_f^2$
2x9'/f	8	10,120.13	1,265.02	$\sigma^2 a/c + 3\sigma^2 c/e + 6\sigma^2 e/f$
1x9'/2x9'	10	1,716.00	171.60	$\sigma^2 a/c + 3\sigma^2 c/e$
1x3'/1x9'	40	2,348.00	58.70	$\sigma^2 a/c$
Total	59	17,619.40		

Number of plants - analysis of variance number 10 - data set 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	3,435.27	3,435.27	$\sigma^2 a/d + 2\sigma^2 d/e + 6\sigma^2 e/f + 30K_f^2$
2x9'/f	8	10,120.13	1,265.02	$\sigma^2 a/d + 2\sigma^2 d/e + 6\sigma^2 e/f$
2x3'/2x9'	20	875.00	43.75	$\sigma^2 a/d + 2\sigma^2 d/e$
1x3'/2x3'	30	3,189.00	106.30	$\sigma^2 a/d$
Total	59	17,619.40		

Total weight - analysis of variance number 7 - data set 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	0.274	0.274	$\sigma^2_{a/c} + 3\sigma^2_{c/e} + 6\sigma^2_{e/f} + 30K_f^2$
2x9'/f	8	48.171	6.021	$\sigma^2_{a/c} + 3\sigma^2_{c/e} + 6\sigma^2_{e/f}$
1x9'/2x9'	10	11.521	1.152	$\sigma^2_{a/c} + 3\sigma^2_{c/e}$
1x3'/1x9'	40	24.246	0.606	$\sigma^2_{a/c}$
Total	59	84.212		

Total weight - analysis of variance number 8 - data set 1

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	0.274	0.274	$\sigma^2_{a/d} + 2\sigma^2_{d/e} + 6\sigma^2_{e/f} + 30K_f^2$
2x9'/f	8	48.171	6.021	$\sigma^2_{a/d} + 2\sigma^2_{d/e} + 6\sigma^2_{e/f}$
2x3'/2x9'	20	16.527	0.826	$\sigma^2_{a/d} + 2\sigma^2_{d/e}$
1x3'/2x3'	30	19.240	0.641	$\sigma^2_{a/d}$
Total	59	84.212		

Total weight - analysis of variance number 9 - data set 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Fields (f)	1	5.275	5.275	$\sigma^2_{a/c} + 3\sigma^2_{c/e} + 6\sigma^2_{e/f} + 30K_f^2$
2x9'/f	8	71.397	8.925	$\sigma^2_{a/c} + 3\sigma^2_{c/e} + 6\sigma^2_{e/f}$
1x9'/2x9'	10	18.081	1.808	$\sigma^2_{a/c} + 3\sigma^2_{c/e}$
1x3'/1x9'	40	21.960	0.549	$\sigma^2_{a/c}$
Total	59	116.713		

Total weight - analysis of variance number 10 - data set 2

Source	Degrees of freedom	Sums of squares	Mean squares	Expected mean square
Field (f)	1	5.275	5.275	$\sigma^2_{a/d} + 2\sigma^2_{d/e} + 6\sigma^2_{e/f} + 30K_f^2$
2x9'/f	8	71.397	8.925	$\sigma^2_{a/d} + 2\sigma^2_{d/e} + 6\sigma^2_{e/f}$
2x3'/2x9'	20	12.829	0.641	$\sigma^2_{a/d} + 2\sigma^2_{d/e}$
1x3'/2x3	30	27.212	0.907	$\sigma^2_{a/d}$
Total	59	116.713		

The independent mean squares in terms of the 1x3' plots are derived from the nested components by the following equations:

$$\hat{\sigma}^2_{1x3'} = \hat{\sigma}^2_{a/c} + \hat{\sigma}^2_{c/e} + \hat{\sigma}^2_{e/f}$$

$$\hat{\sigma}^2_{1x9'} = \frac{\hat{\sigma}^2_{a/c}}{3} + \hat{\sigma}^2_{c/e} + \hat{\sigma}^2_{e/f}$$

$$\hat{\sigma}^2_{2x9'} = \frac{\hat{\sigma}^2_{a/e}}{6} + \frac{\hat{\sigma}^2_{c/e}}{2} + \hat{\sigma}^2_{e/f},$$ for analyses 7 and 9, and

$$\hat{\sigma}^2_{1x3'} = \hat{\sigma}^2_{a/d} + \hat{\sigma}^2_{d/e} + \hat{\sigma}^2_{e/f}$$

$$\hat{\sigma}^2_{2x3'} = \frac{\hat{\sigma}^2_{a/d}}{2} + \hat{\sigma}^2_{d/e} + \hat{\sigma}^2_{e/f}$$

$$\hat{\sigma}^2_{2x9'} = \frac{\hat{\sigma}^2_{a/d}}{6} + \frac{\hat{\sigma}^2_{d/e}}{3} + \hat{\sigma}^2_{e/f}$$ for analyses 8 and 10.

Estimated variances within fields are shown in Tables 14 and 15.

Table 14.--Estimated variances of number of plants within fields

Analysis and data set	Nested component	Estimate	Independent mean square	Estimate 1/
Number 7	$\sigma^2_{a/c}$	100.37	σ^2_a	255.50
	$\sigma^2_{c/e}$	0.60	σ^2_c	188.59
Data set 1	$\sigma^2_{e/f}$	154.53	σ^2_e	171.56
Number 8	$\sigma^2_{a/d}$	91.20	σ^2_a	255.50
	$\sigma^2_{d/e}$	11.91	σ^2_d	209.91
Data set 1	$\sigma^2_{e/f}$	152.39	σ^2_e	171.56
Number 9	$\sigma^2_{a/c}$	58.70	σ^2_a	278.57
	$\sigma^2_{c/e}$	37.63	σ^2_c	239.44
Data set 2	$\sigma^2_{e/f}$	182.24	σ^2_e	210.84
Number 10	$\sigma^2_{a/d}$	106.30	σ^2_a	278.57
	$\sigma^2_{d/e}$	31.28	σ^2_d	255.42
Data set 2	$\sigma^2_{e/f}$	203.55	σ^2_e	210.84

1/ Mean squares are adjusted to the 1 bed x 3 foot plot level.

Table 15.--Estimated variance of total weight within fields

Analysis and data set	Nested component	Estimate	Independent mean square	Estimate 1/
Number 7	$\sigma^2_{a/c}$	0.606	σ^2_a	1.60
	$\sigma^2_{c/e}$	0.182	σ^2_c	1.20
Data set 1	$\sigma^2_{e/f}$	0.811	σ^2_e	1.00
Number 8	$\sigma^2_{a/d}$	0.641	σ^2_a	1.60
	$\sigma^2_{d/e}$	0.092	σ^2_d	1.28
Data set 1	$\sigma^2_{e/f}$	0.866	σ^2_e	1.00
Number 9	$\sigma^2_{a/c}$	0.549	σ^2_a	2.16
	$\sigma^2_{c/e}$	0.420	σ^2_c	1.79
Data set 2	$\sigma^2_{e/f}$	1.186	σ^2_e	1.49
Number 10	$\sigma^2_{a/d}$	0.907	σ^2_a	2.16
	$\sigma^2_{d/e}$	-0.133	σ^2_d	1.61
Data set 2	$\sigma^2_{e/f}$	1.381	σ^2_e	1.49

1/ Mean squares are adjusted to the 1 bed x 3 foot plot level.

Since the difference between data set 1 and 2 is merely a distance of the width of a bed in locating the corner of a 2x9' plot, it seems reasonable to average the two data sets' estimates of the independent mean squares. The averages of the data sets' estimates are shown in Table 16.

Table 16.--Estimated independent mean squares for four plot sizes and shapes

Plot	Independent mean square	Number of plants	Total weight
1x3'	$\hat{\sigma}^2{}_a$	267.04	1.88
1x9'	$\hat{\sigma}^2{}_c$	214.02	1.50
2x3'	$\hat{\sigma}^2{}_d$	217.66	1.44
2x9'	$\hat{\sigma}^2{}_e$	191.20	1.24

These independent mean squares can be considered relative to the variance of the 1x3' plot. The ratios are shown in Table 17.

Table 17.--Ratios of estimated variances to estimated variance of the 1x3' plot

Plot	Ratio	Number of plants	Total weight
1x3'	$\hat{\sigma}^2{}_a/\hat{\sigma}^2{}_a$	1.00	1.00
1x9'	$\hat{\sigma}^2{}_c/\hat{\sigma}^2{}_a$	0.80	0.80
2x3'	$\hat{\sigma}^2{}_d/\hat{\sigma}^2{}_a$	0.82	0.77
2x9'	$\hat{\sigma}^2{}_e/\hat{\sigma}^2{}_a$	0.72	0.66

By substitution into equation (1) on page 12, we have

$$C_{2x3'} = W\left(\frac{T_R + .3\sqrt{\ell} = 2\sqrt{w}}{V} + T_B + 5w\sqrt{\ell}\right)$$

$$= W\left(\frac{10.00 + 3.34}{3} + 2.00 + 17.30\right) = 23.75\ W$$

and $$C_{2x9'} = W\left(\frac{10.00 + 3.72}{3} + 2.00 + 30.00\right) = 36.57\ W.$$

Thus, $\frac{C_a}{C_a} = 1.00$, $\frac{C_c}{C_a} = \frac{21.30W}{14.82W} = 1.44$, $\frac{C_d}{C_a} = \frac{23.75W}{14.82W} = 1.60$

and $\frac{C_e}{C_a} = \frac{36.57W}{14.82W} = 2.47.$

The relative cost variances are shown in Table 18.

Table 18.--Relative cost variances for four plot sizes and shapes

Plot	Ratio	Number of plants	Total weight
1x3'	$\hat{\sigma}^2_a/\hat{\sigma}^2_a \times C_a/C_a$	1.00	1.00
1x9'	$\hat{\sigma}^2_c/\hat{\sigma}^2_a \times C_c/C_a$	1.15	1.15
2x3'	$\hat{\sigma}^2_d/\hat{\sigma}^2_a \times C_d/C_a$	1.31	1.23
2x9'	$\hat{\sigma}^2_e/\hat{\sigma}^2_a \times C_e/C_a$	1.78	1.63

Table 18 shows that the 1x3' plot is the optimum plot size of these four plots for estimating number of plants and total weight.

4. Selection of the Optimum Plot

The relative cost variances for all eight plot sizes and shapes can now be compared relative to the 1x3' plot. From Tables 8, 13, and 18, it is seen that for plant counts the 1x3' plot is the optimum with the 1x6'plot second best. For estimating total weight the 1x6' plot is the best followed by the 1x3' plot. It is also noted that the three one bed plots are nearer the optimum than the other five plots. Of course, these results indicate only the optimum of the eight plots studied. A 1x4' or 1x5' plot would be near the optimum for estimating both number of plants and total weight. Considering both the discrete procedure and the procedure illustrated in the Appendix, the optimum plot is one bed wide and three to five feet long.

If a more precise indication of the size of the optimum plot is required, additional data should be collected to obtain variance estimates for plots near the 1x3' and 1x6' size. Additional data on costs would also be desirable. Because the cost function is based primarily upon judgement and variances are estimated, it seems inadvisable to attempt any greater precision with the data available. However, a method of selecting the optimum length of a plot one bed wide is illustrated in the Appendix. This method is a combination of the discrete and the continuous procedures because the optimum width is determined to be one bed by the discrete method.

APPENDIX

Illustration of a Method of Selecting the Optimum Length of a Plot One Bed Wide for Estimating Carrot Yield

The variability of crop yields from unit to unit in experimental fields has been studied extensively for units of different sizes. A study by H. F. Smith[1/] indicates that variance of a plot of k basic units, on a basic unit basis, in a field is given by:

$$\sigma^2_k = \frac{\sigma^2}{k^b} \text{ or } \log \sigma^2_k = \log \sigma^2 - b \log k.$$

Here, σ^2_k is the variance within a field, on a basic unit basis, of a plot containing k basic units, σ^2 is the variance of a plot containing a single basic unit and b is an index of soil heterogeneity.

If this functional relationship between plot size and variance is accepted and if a good estimate of b is available, then an estimate of σ^2 is sufficient to permit estimation of the variance of plots within a reasonable size difference from the size of the basic unit plot. If a good estimate of b is not available, then the equation $b = (\log \sigma^2 - \log \sigma^2_k)/\log k$ can be used to derive an estimate of b. In fact, there will be a P-1 equations estimating b for P = 1, 2, 3, ... plot sizes for which an estimate of variance is available. These estimates can be combined for an improved estimate of b.

From the carrot data $\hat{\sigma}^2_{1x3'} = 274.24$, $\hat{\sigma}^2_{1x6'} = 248.32$ and $\hat{\sigma}^2_{1x9'} = 213.29$ are averages of various estimates for each size of plot for number of plants. For total weight the averages are $\hat{\sigma}^2_{1x3'} = 1.84$, $\hat{\sigma}^2_{1x6'} = 1.37$ and $\hat{\sigma}^2_{1x9'} = 1.46$. Thus, estimates of b for number of plants are:

$$\hat{b}_1 = \frac{\log \hat{\sigma}^2_{1x3'} - \log \hat{\sigma}^2_{1x9'}}{\log (\text{No. } 1x3' \text{ in } 1x9')} = \frac{\log 274.24 - \log 213.29}{\log 3} = .2289$$

1/ Smith, H. F. "An Empirical Law Describing Heterogeneity in Yields of Agricultural Crops". Journal of Agricultural Science, Volume 28, p. 1-23, 1938.

$$\hat{b}_2 = \frac{\log \hat{\sigma}^2_{1x3'} - \log \hat{\sigma}^2_{1x6'}}{\log \text{(No. 1x3' in 1x6')}} = \frac{\log 274.24 - \log 248.32}{\log 2} = .1435.$$

For total weight the estimates are:

$$\hat{b}_3 = \frac{\log \hat{\sigma}^2_{1x3'} - \log \hat{\sigma}^2_{1x9'}}{\log \text{(No. 1x3' in 1x9')}} = \frac{\log 1.84 - \log 1.37}{\log 3} = .2104,$$

$$\hat{b}_4 = \frac{\log \hat{\sigma}^2_{1x3'} - \log \hat{\sigma}^2_{1x6'}}{\log \text{(No. 1x3' in 1x6')}} = \frac{\log 1.84 - \log 1.46}{\log 2} = .4256.$$

One method of obtaining a single estimate of b is to use

$\hat{b} = .46 \left(\frac{\hat{b}_1 + \hat{b}_2}{2} \right) + .54 \left(\frac{\hat{b}_3 + \hat{b}_4}{2} \right) = .2962.$ This weights the estimates of b for number of plants and total weight approximately in proportion to the strength 1/ of their relationship to yields.

Using b = .2962, we have $\hat{\sigma}_K^2 = \hat{\sigma}^2 / (K)^{.2962}$ or $\log \hat{\sigma}^2_K = \log \hat{\sigma}^2 - .2962 \log K$.

For the carrot data the 1x3' plot is the basic unit and the unit of interest is the 1x ℓ' plot, where ℓ is the length of the plot in feet. The range of values for ℓ should not be much beyong the plot sizes for which data is available, that is, 3 to 9 feet. In terms of the carrot data, $\log \sigma^2_{1x\ell'} = \log \sigma^2_{1x3'} - .2962 \log \left(\frac{\ell'}{3'} \right)$. The estimated variances for one bed plots one foot to nine feet in length are shown in Table 19.

1/ Strength is measured in terms of correlation coefficients for the 1x3' size plot.

Table 19.--Estimated variance of selected plot sizes 1/

Plot length (feet)	Number of plants	Total weight
1	399.75	2.55
2	309.29	2.07
3	(274.24) 274.24	(1.84) 1.84
4	251.88	1.69
5	235.78	1.58
6	(248.32) 223.37	(1.37) 1.50
7	213.40	1.43
8	205.10	1.38
9	(213.29) 198.09	(1.46) 1.33

1/ Figures shown in parentheses are the estimated variances used in estimating b_1, b_2, b_3, and b_4. The functional relationship of variance on plot length does not seem to fit the carrot data well.

The estimated variance of these plots can be considered relative to the variance of the basic unit, the 1x3' plot. The ratio of the variance of a plot to the variance of a 1x3' plot indicates the number of plots of that size required per field to give as precise a field estimate as would one 1x3' plot. The ratios are shown in Table 20.

Table 20.--Ratio of estimated variances to the variance of the 1x3' plot of selected plot sizes

Plot length (feet)	Number of plants	Total weight
1	1.46	1.39
2	1.13	1.12
3	1.00	1.00
4	.92	.92
5	.86	.86
6	.81	.82
7	.78	.78
8	.75	.75
9	.72	.72

The cost function $C_p = W\left(\frac{T_R + .3\sqrt{\ell} + 2\sqrt{w}}{V} + T_B + 5w\sqrt{\ell}\right)$

can be used to estimate the cost of each one bed plot from 1 to 9 feet in length.

The form of the cost function for these one bed plots is:

$$C_{1x\ell'} = W\left(\frac{10.00 + .3\sqrt{\ell} + 2.00}{3} + 2.00 + 5\sqrt{\ell}\right)$$

$$= W\ (4.00 + .1\sqrt{\ell} + 2.00 + 5\sqrt{\ell}) = W\ (6.00 + 5.1\sqrt{\ell}).$$

The costs can be compared relative to the basic unit, by calculating the ratio of the cost of each plot to the cost of the 1x3' plot. The cost data are presented in Table 21.

Table 21.--Estimated cost and ratio to cost of the 1x3' plot for selected plot sizes

Plot length	Cost of plot 1/	Ratio of cost of 1x3' plot
(feet)		
1	11.10 W = \$0.48	.75
2	13.19 W = \$0.57	.89
3	14.82 W = \$0.64	1.00
4	16.20 W = \$0.70	1.09
5	17.42 W = \$0.75	1.18
6	18.50 W = \$0.80	1.25
7	19.52 W = \$0.85	1.32
8	20.43 W = \$0.89	1.38
9	21.30 W = \$0.92	1.44

1/ No assumption is necessary for W. However, if W is assumed at 4 1/3 cents/minute or \$2.60/hour, the costs shown result.

The relative cost variances, which enable determination of the optimum length of a one bed plot, are shown in Table 22.

Table 22.--Relative cost variance for selected plot sizes

Plot length	Number of plants	Total weight
(Feet)		
1	1.10	1.04
2	1.01	1.00
3	1.00	1.00
4	1.00	1.00
5	1.01	1.01
6	1.01	1.02
7	1.03	1.03
8	1.04	1.04
9	1.04	1.04

The length of plot which will minimize the relative cost variance can also be found by taking the derivative of the relative cost variance with respect to length, setting it equal to zero and solving for length. The second derivative with respect to length can then be used to demonstrate that the length solved for does minimize the relative cost variance. Relative cost variance is:

$$\frac{\hat{\sigma}^2_{1x\ell'}}{\hat{\sigma}^2_{1x3'}} \times \frac{C_{1x\ell'}}{C_{1x3'}} = \frac{\hat{\sigma}^2_{1x3'} \Big/ \left(\frac{\ell'}{3'}\right)^{.2962}}{\sigma^2_{1x3'}} \times \frac{W\ (6.0 + 5.1\ \sqrt{\ell'})}{W\ (14.82)}$$

$$= \frac{(3)^{.2962}}{14.82} \left[6.0\ (\ell)^{-.2962} + 5.1\ (\ell)^{.2038}\right]$$

The first derivative is:

$$\frac{(3)^{.2962}}{14.82} \left[(6.0)\ (-.2962)\ (\ell)^{-1.2962} + (5.1)\ (.2038)\ (\ell)^{-.7962}\right]$$

$$\text{or} \quad \frac{(3)^{.2962}}{14.82} \left[-1.7772\ (\ell)^{-1.2962} + 1.3094\ (\ell)^{-.7962}\right]$$

Setting the derivative equal to zero, then

$$1.0394\ (\ell)^{-.7962} = 1.7772\ (\ell)^{-1.2962}$$

or $$\frac{(\ell)^{-.7962}}{(\ell)^{-1.2962}} = (\ell)^{.5000} = \frac{1.7772}{1.0394} = 1.71 \text{ or } \ell = (1.71)^2 = 2.92 \text{ feet.}$$

The second derivative is:

$$\frac{(3)^{.2962}}{14.82}\left[(1.7772)\ (-1.2962)\ (\ell)^{-2.2962} + (1.0394)\ (-.7962)(\ell)^{-1.7962}\right]$$

or $$\frac{(3)^{.2962}\ (\ell)^{-1.7962}}{14.82}\left[\frac{2.3036}{(\ell)^{.5000}} - .8276\right] > 0, \text{ for } \ell = 2.92 \text{ feet.}$$

Thus, the relative cost variance is a minimum for ℓ = 2.92 feet. This result is consistent with Table 22.

The continuous approach to the determination of optimum plot size, illustrated here, indicates the 1x3' or 1x4' plot is optimum. Any plot one bed in width and from 2 to 5 feet in length seems to be near the optimum. It should be noted that the one and two foot plots are an extrapolation from the range of the data. In this illustration the functional relationship does not fit the data very well. This method would be an adequate procedure when more data are available, the cost function is well known and the relationship fits the data. The continuous method yields a more specifically defined optimum plot and provides a more detailed indication of the degree of flatness near the optimum than the discrete procedure.